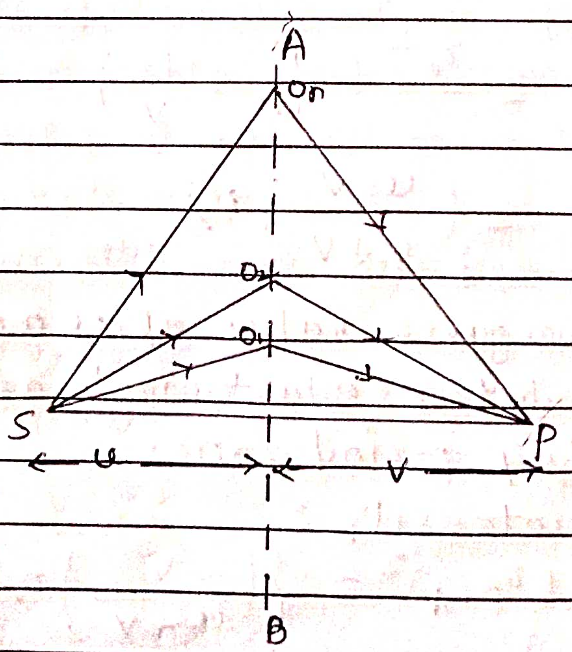
Q. (a) Describe and discuss the construction of a Zone plate.

(b) Derive an expression for its focal length when it acts like a convergent lens.

(c) How it acts like a divergent lens?

Ans. (a) Zone plate : A zone plate is a specially constructed screen such that light is abstructed from every alternate zone. It consists of a glass plate having concentric circles of radoi proportional to the square root of natural numbers 1, 2, 3, ··· Now even or odd annular spaces between the two circles are blackened to make them apaque.

We consider AB as a zone plate placed perpendicular to the plane of fig. S be a monochromatic light Source of wavelength λ having prependicular distance SO = u from the Zone plate. Let we have to determine resultant intensity at P, where OP = V.



Now we take points O_1, O_2 ··· O_n on the plate such that

$SO_1 + O_1P = SO + OP + \frac{\lambda}{2} = u + v + \frac{\lambda}{2}$

$SO_2 + O_2P = SO + OP + 2\frac{\lambda}{2} = u + v + 2\frac{\lambda}{2}$

And. $SO_n + O_nP = SO + OP + n\frac{\lambda}{2} = u + v + n\frac{\lambda}{2}$

Now. we draw concentric circles of radii $OO_1 = r_1$, $OO_2 = r_2$, ..., $OO_n = r_n$ on the plate

Now, $SO_n^2 = SO^2 + OO_n^2$

$\therefore SO_n = (u^2 + r_n^2)^{1/2} = u\left(1 + \frac{r_n^2}{u^2}\right)^{\frac{1}{2}}$

$= u\left(1 + \frac{1}{2}\frac{r_n^2}{u^2}\right) \left[As, \frac{r_n^2}{u^2} << 1\right]$

Similarly, $PO_n = v\left[1 + \frac{1}{2}\frac{r_n^2}{v^2}\right]$

$\therefore SO_n + O_nP = u + v + \frac{n\lambda}{2} = u\left(1 + \frac{1}{2}\frac{r_n^2}{u^2}\right) + v\left(1 + \frac{1}{2}\frac{r_n^2}{v^2}\right)$

$= u + v + \frac{1}{2} r_n^2 \left(\frac{1}{u} + \frac{1}{v}\right)$

or, $\frac{n\lambda}{2} = \frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right)$

$\therefore r_n^2 = \left(\frac{u \cdot v}{u+v}\right) n\lambda$ ——— (1)

Thus for a given value of u and v, $r_n \propto \sqrt{n}$

If, A_n = Area between last circle and last but one circle = nth half period zone.

Resultant intensity :

$A_n = \pi r_n^2 - \pi (r_{n-1})^2 = \left\{\left(\frac{uv}{u+v}\right) n\lambda - \left(\frac{uv}{u+v}\right)(n-1)\lambda\right\}$

$= n\lambda\left(\frac{uv}{u+v}\right)$

Thus A_n is independent of n. Hence areas of al annular zones are equal. Thus numerical values of displacements R_1, R_2 ..... due to 1st, 2nd ... zone diminish slowly with increase of n. As the displacement from alternate zones will have opposite phases, the resultant displacement R at P is given by –

$$R = R_1 - R_2 + R_3 - \cdots$$

$$= \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \left(\frac{R_3}{2} - R_4 - \frac{R_5}{2}\right) + \cdots$$

$$= \frac{R_1}{2}, \text{ when n is very large.}$$

If we stop the secondary waves from second, fourth, sixth, ... zones then, $R = R_1 + R_3 + R_5 + \cdots$ which is many times greater than the resultant displacement due to all zones.

So to construct a zone plate it is only necessary to draw on a white paper, a large number of concentric circles with radii proportional to square roots of natural numbers and then to blacken the latemate zones. A reduce photograph on a glass plate of this drawing constitute a zone plate.

(b) Zone plate as a convergent lens:

from equation (1) radius of the nth zone is given by –

$$r_n^2 = \left(\frac{u \cdot v}{u+v}\right) n\lambda$$

$$\therefore \frac{u+v}{u \cdot v} = \frac{n\lambda}{r_n^2}$$

$$\text{or. } \frac{1}{v} + \frac{1}{u} = \frac{n\lambda}{r_n^2}$$

This expression is constant for a given zone plate. If we compare it withe the convex lens equation for a real image

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get, $\frac{1}{f_n} = \frac{n\lambda}{r_n^2}$ $\therefore$ $f_n = \frac{r_n^2}{n\lambda}$ — (11)

Thus a zone plate acts as a converging lens. A zone plate has a number of foci which depend on the number of zones used as well as the wavelength of light employed.

If n is ~~each~~ even, each clear is occupied by an even number of zones which concle in pairs; but if n is odd there is maximum intensity at p. Thus a zone plate gives a succesion of images corresponding to focal length f, $\frac{f}{3}$, $\frac{f}{5}$ etc where $u = \infty$ for a plane wave

(c) In n be given odd negative values in equation (12) then even with a distant source, virtual images are formed corresponding to focal lenghts $-f, -f/3, -f/5$ etc. for a zone plate acts like a circular diffraction grating of unequal spacing and wavelets issuning out may be considered to arise from points on left of zone plate. The rays which meet at points p may be considered as diffracted rays, but other side of line $O_1 P$. It means in upward direction. These will give rise to a series of virtual images on the side of zone plate in which s is situated. Thus a zone plate acts like a divergent lens.